Acta Oceanol. Sin., 2018, Vol. 37, No. 7, P. 55–61
DOI: 10.1007/s13131-018-1243-z
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The dispersion of surface contaminants by Stokes drift in random waves

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Received 8 July 2017; accepted 10 October 2017

Abstract
Contaminants that are floating on the surface of the ocean are subjected to the action of random waves. In the literature, it has been asserted by researchers that the random wave action will lead to a dispersion mechanism through the induced Stokes drift, and that this dispersion mechanism may have the same order of significance comparable with the others means due to tidal currents and wind. It is investigated whether or not surface floating substances will disperse in the random wave environment due to the induced Stokes drift. An analytical derivation is first performed to obtain the drift velocity under the random waves. From the analysis, it is shown that the drift velocity is a time-independent value that does not possess any fluctuation given a specific wave energy spectrum. Thus, the random wave drift by itself should not have a dispersive effect on the surface floating substances. Experiments were then conducted with small floating objects subjected to P-M spectral waves in a laboratory wave flume, and the experimental results reinforced the conclusion drawn.

Key words: dispersion, surface contaminants, random wave, Stokes drift

Citation: Huang Guoxing, Law Wing-Keung Adrian. 2018. The dispersion of surface contaminants by Stokes drift in random waves. Acta Oceanologica Sinica, 37(7): 55–61, doi: 10.1007/s13131-018-1243-z

1 Introduction

Floating contaminants on the surface of the ocean, such as an oil patch, are subjected to a number of dispersion mechanisms, including ambient turbulence (Maksimenko, 1990; Craig and Banner, 1994; Huang et al., 2013) as well as shear dispersion due to winds, ocean currents and tidal flows (Sanderson and Pall, 1990; Mantovanelli et al., 2012). Through the dispersive action, the bulk planar size of the surface contaminants would then enlarge with time. When a small size scale of up to a few kilometers is considered, Herterich and Hasselmann (1982) suggested that the action of the random waves is a possible mechanism that has the same order of significance comparable with the others. They explained that the dispersion mechanism by waves is through the random fluctuations of the wave-induced Stokes drift. Subsequently, most of the recent investigations on the wave-induced dispersion of floating contaminants on the sea surface, i.e., Mesquita et al. (1992), Giarrusso et al. (2001), Buick et al. (2001) and Pugliese Carratelli et al. (2011), were grounded in this theoretical development by Herterich and Hasselmann (1982).

The detailed derivation of Herterich and Hasselmann (1982) can be summarized as follows. With a random wave field in deep water, the ensemble-mean Stokes drift velocity can be expressed as

$$< u_{\rm s}(z) >= 2\int_0^\infty f(k)\omega k\, {\rm e}^{2kz}\,{\rm d}\,k, \tag{1}$$

where $< u_{\rm s}(z) >$ is the mean drift velocity with the angle brackets denoting the ensemble averaging, $f(k)$ is the wave number spectrum; $\omega=\sqrt{gk}$ is the angular frequency, k is the wave number; and z is the vertical co-ordinate measured positively upward from the mean water level.

Herterich and Hasselmann (1982) suggested that this ensemble mean represents the time-average drift velocity, and that an individual particle at different positions under the wavy profile will undergo an "instantaneous" drift velocity, $u_{\rm s}$, such that the particle will experience drift-velocity fluctuations $u'_{\rm s}$ ($u'_{\rm s}=u_{\rm s}-< u_{\rm s} >$) due to the fluctuations of the local Rayleigh-distributed wave amplitudes in the random sea. The random fluctuations in the drift velocity would then translate to a dispersion effect following the random-walk theory, and the corresponding dispersion coefficient, D , can be obtained as

$$D =< {u'_{\rm s}}^2 > \tau, \tag{2}$$

where τ represents the integral autocorrelation time of the fluctuations. Herterich and Hasselmann (1982) further extended the concept to a multi-directional sea, in which case the following dispersion tensors were obtained.

$$\begin{pmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{pmatrix} = \frac{\pi}{4g^2}\int_0^\infty \omega^6 f^2(\omega)\,{\rm d}\,\omega \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \Theta(\theta_1)\Theta(\theta_2)\times [1+\cos(\theta_1-\theta_2)]^2 \begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix} {\rm d}\theta_1{\rm d}\theta_2. \tag{3}$$

Foundation item: The State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering Research Foundation of China under contract No. 2015491311.
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where $\Theta(\theta)$ is a frequency-independent directional spreading factor; D_{ij} a diffusion tensor; $A_1=(\cos\theta_1+\cos\theta_2)^2$; $A_2=(\cos\theta_1+\cos\theta_2)(\sin\theta_1+\sin\theta_2)$; $A_3=(\cos\theta_1+\cos\theta_2)(\sin\theta_1+\sin\theta_2)$; and $A_4=(\sin\theta_1+\sin\theta_2)^2$.

Buick et al. (2001) presented the experimental results for the case of a pair of surface particles in a random wave basin to verify the theoretical predictions by Herterich and Hasselmann (1982). They show that the experimental results appear to agree with the theoretical prediction at lower values of the particle separating distance. However, the measurements deviated substantially with the prediction when the distance increased.

Herterich and Hasselmann (1982) theoretical development is based on the conjecture that the Stokes drift velocity fluctuates in a random manner under the multi directional random sea. In order to address the issue, it is thus essential to review the fundamental concepts related to the analysis of random waves. It is well known that there are two possible approaches to analyze the time series of a random wave field, namely the time-domain and frequency-domain analysis (US Army Corps of Engineers, 2002). In the time-domain analysis, or commonly known also as the zero-crossing approach, the time series of the surface elevation record is a composite of a number of individual waves which have different wave heights and periods based on the zero-crossing intervals. Longuet-Higgins (1952) shows that the probability density of the individual zero-crossing wave height follows the Rayleigh distribution. On the other hand, for the frequency-domain analysis or also commonly known as the spectral analysis, the surface elevation is expanded in the form of a Fourier series:

$$\begin{aligned}\eta(t)&=\sum_{i=1}^{\infty}a_i\cos(k_ix-\omega_it+\varepsilon_i)\\&=\sum_{i=1}^{\infty}\sqrt{2s(\omega_i)\Delta\omega_i}\cos(k_ix-\omega_it+\varepsilon_i),\end{aligned}\tag{4}$$

where a_i, k_i, ω_i and ε_i are the amplitude, wave number, angular frequency and random phase of each wave component, respectively; and $s(\omega)$ is the frequency spectrum. The functional form implies that the time series is the superposition of an infinite number of linear waves with different frequencies. The two different approaches are typically used independently, although it has been shown that both approaches give a similar value of a significant wave height in deep waters (i.e., Gōda, 2010).

In Herterich and Hasselmann (1982) approach, the ensemble-mean Stokes drift velocity is first obtained in the frequency-domain, and then the 'instantaneous' fluctuating drift velocity is evaluated in the time-domain. This cross over poses a potential ambiguity in the derived relationship for the dispersion effect. Here, the question of whether or not the drift velocity of the random surface waves would lead to a dispersion effect of surface floating substances needs to be reexamined. In the following, we shall show that the drift velocity obtained in the frequency domain is a time-independent value that can be considered as an instantaneous value as well, such that the random fluctuations would not exist. An experimental study will also be presented to reinforce the conclusion.

2 Stokes drift in regular and random waves

2.1 *Stokes drift in regular waves*

The Stokes drift in regular waves is first derived before the drift under the random waves is considered. The velocity potential in a progressive regular wave is (e.g., Dean and Dalrymple, 1991) well understood as

$$\begin{aligned}\varphi(x,z,t)&=\varphi^{(1)}(x,z,t)+\varphi^{(2)}(x,z,t)+\ldots\\&=\frac{a\omega}{k}\left[\frac{\cosh k(z+d)}{\sinh kd}\sin\theta+\right.\\&\left.\frac{3}{8}ak\frac{\cosh 2k(z+d)}{\sinh^4 kd}\sin 2\theta\right],\end{aligned}\tag{5}$$

where d is the water depth. The Eulerian velocity and displacement up to second order can then be derived as

$$\begin{aligned}u(x,z,t)&=\frac{\partial\varphi}{\partial x}=\frac{\partial(\varphi^{(1)}+\varphi^{(2)})}{\partial x}\\&=a\omega\left[\frac{\cosh k(z+d)}{\sinh kd}\cos\theta+\right.\\&\left.\frac{3}{4}ak\frac{\cosh 2k(z+d)}{\sinh^4 kd}\cos 2\theta\right],\end{aligned}\tag{6}$$

$$\begin{aligned}w(x,z,t)&=\frac{\partial\varphi}{\partial z}=\frac{\partial(\varphi^{(1)}+\varphi^{(2)})}{\partial z}\\&=a\omega\left[\frac{\sinh k(z+d)}{\sinh kd}\sin\theta+\right.\\&\left.\frac{3}{4}ak\frac{\sinh 2k(z+d)}{\sinh^4 kd}\sin 2\theta\right],\end{aligned}\tag{7}$$

$$x-x_0=\int_0^t u\,\mathrm{d}t\approx -a\frac{\cosh k(z_0+d)}{\sinh kd}\sin\theta_0,\tag{8}$$

$$z-z_0=\int_0^t w\,\mathrm{d}t\approx a\frac{\cosh k(z_0+d)}{\sinh kd}\cos\theta_0,\tag{9}$$

where $\theta=kx-\omega t$, and (x_0,z_0) is the initial position.

Given the Eulerian velocities and displacements, the Lagrangian horizontal velocity can be obtained by Taylor's expansion as

$$\begin{aligned}u(x,z,t)&=u(x_0,z_0,t)+\left(\frac{\partial u}{\partial x}\right)_{x_o,\,z_o}(x-x_0)+\\&\left(\frac{\partial u}{\partial z}\right)_{x_o,\,z_o}(z-z_0)+O\left((ka)^3\right);\\&=c\,(ka)\frac{\cosh k(z_0+d)}{\sinh kd}\cos\theta_0+\frac{c(ka)^2}{2\sinh^2 kd}\times\\&\left[\frac{3\cosh 2k(z_0+d)}{2\sinh^2 kd}-1\right]\cos 2\theta_0+\\&c(ka)^2\frac{\cosh 2k(z_0+d)}{2\sinh^2 kd}+O\left((ka)^3\right).\end{aligned}\tag{10}$$

From Eq. (10), it can also be concluded that, up to the second order, the Lagrangian velocity consists of a first order and second order oscillatory components plus a time mean Stokes drift component of

$$u_{\mathrm{s}}=c(ka)^2\frac{\cosh 2k(z_0+d)}{2\sinh^2 kd}.\tag{11}$$

The above derivations and results are well known and can be commonly found in the literature. We show the derivation here in details to illustrate the fact that the drift velocity appears as a component to the instantaneous velocity, and that no period averaging is needed to obtain the relationship in Eq. (11). Hence, the drift velocity is a time-independent quantity that can also be considered as an instantaneous value, or in other words, it does not possess any fluctuations. Note that similar derivations can be performed for the z-direction to illustrate that such drift velocity does not exist vertically, as expected.

2.2 *Stokes drift in random waves*

A spectral or superposition method, which treats the surface profile as a superposition of infinitude linear waves with different frequencies, amplitudes and phases, can be used to analyze a random wave field. Here, we shall first derive the interaction between two sinusoidal wave components. Subsequently, a solution will be generalized to include the entire wave spectrum.

The surface boundary condition stipulates that the second order velocity potential of a progressive wave (Dean and Dalrymple, 1991) should satisfy the following:

$$\frac{\partial^2\varphi^{(2)}}{\partial t^2}+g\frac{\partial\varphi^{(2)}}{\partial z}=-2\left(\frac{\partial\varphi^{(1)}}{\partial x}\frac{\partial^2\varphi^{(1)}}{\partial x\partial t}+\frac{\partial\varphi^{(1)}}{\partial z}\frac{\partial^2\varphi^{(1)}}{\partial z\partial t}\right)+\frac{1}{g}\frac{\partial\varphi^{(1)}}{\partial t}\left(g\frac{\partial^2\varphi^{(1)}}{\partial z^2}+\frac{\partial^3\varphi^{(1)}}{\partial z\partial t^2}\right). \tag{12}$$

Taking two distinct-frequency sinusoidal waves along the x-axis, the first order velocity potential due to the superposition is

$$\varphi^{(1)}(x,z,t)=\frac{ga_1}{\omega_1}\frac{\cosh k_1(z+d)}{\cosh k_1 d}\sin(k_1x-\omega_1t)+\frac{ga_2}{\omega_2}\frac{\cosh k_2(z+d)}{\cosh k_2 d}\sin(k_2x-\omega_2t). \tag{13}$$

Substituting Eq. (13) into Eq. (12) and on the surface, z=0, we have

$$\begin{aligned}\frac{\partial^2\varphi^{(2)}}{\partial t^2}+g\frac{\partial\varphi^{(2)}}{\partial z}=&\frac{1}{2}\sin 2\theta_1\left[3\frac{g^2{a_1}^2{k_1}^2}{\omega_1}(\tanh^2 k_1d-1)\right]+\\&\frac{1}{2}\sin 2\theta_2\left[3\frac{g^2{a_2}^2{k_2}^2}{\omega_2}(\tanh^2 k_2d-1)\right]+\\&\frac{1}{2}\sin(\theta_1+\theta_2)\left[-2g^2a_1a_2k_1k_2\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)+\right.\\&2g^2a_1a_2k_1k_2\tanh k_1d\tanh k_2d\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)-g^2a_1a_2\times\\&\left.\left(\frac{{k_1}^2}{\omega_1}+\frac{{k_1}^2}{\omega_2}\right)+ga_1a_2(k_1\omega_1\tanh k_1d+k_2\omega_2\tanh k_2d)\right]+\\&\frac{1}{2}\sin(\theta_1-\theta_2)\left[-2g^2a_1a_2k_1k_2\left(\frac{1}{\omega_1}-\frac{1}{\omega_2}\right)+\right.\\&2g^2a_1a_2k_1k_2\tanh k_1d\tanh k_2d\left(\frac{1}{\omega_1}-\frac{1}{\omega_2}\right)-\\&g^2a_1a_2\left(\frac{{k_1}^2}{\omega_1}-\frac{{k_1}^2}{\omega_2}\right)+ga_1a_2\times\\&\left.(k_1\omega_1\tanh k_1d-k_2\omega_2\tanh k_2d)\right],\end{aligned} \tag{14}$$

where $\theta_1=k_1x-\omega_1t$, and $\theta_2=k_2x-\omega_2t$.

Denote a function G where by:

$$\begin{aligned}G^{\pm}(\omega_1,\omega_2)=&\,g_{12}^{\pm}\pm g_{21}^{\pm}\\=&-g^2\left[k_1k_2\left(\frac{1}{\omega_1}\pm\frac{1}{\omega_2}\right)\times\right.\\&(1\mp\tanh k_1d\tanh k_2d)+\\&\left.\frac{1}{2}\left(\frac{{k_1}^2}{\omega_1\cosh^2 k_1d}\pm\frac{{k_2}^2}{\omega_2\cosh^2 k_2d}\right)\right].\end{aligned} \tag{15}$$

Then

$$\begin{aligned}\frac{\partial^2\varphi_2}{\partial t^2}+g\frac{\partial\varphi_2}{\partial z}=&\,a_1a_2G^{\pm}(\omega_1,\omega_2)\sin(k_{\pm}x-\omega_{\pm}t)+\\&\frac{1}{2}{a_1}^2G^{+}(\omega_1,\omega_1)\sin 2(k_1x-\omega_1t)+\\&\frac{1}{2}{a_2}^2G^{+}(\omega_2,\omega_2)\sin 2(k_2x-\omega_2t),\end{aligned} \tag{16}$$

where $k_{\pm}=k_1\pm k_2$; and $\omega_{\pm}=\omega_1\pm\omega_2$.

Consider a special solution:

$$\begin{aligned}\varphi^{(2)}(x,z,t)=&\,a_1a_2E_{12}^{\pm}\frac{\cosh k_{\pm}(z+d)}{\cosh k_{\pm}d}\times\\&\sin(k_{\pm}x-\omega_{\pm}t)+\frac{1}{2}{a_1}^2E_{11}^{\pm}\times\\&\frac{\cosh 2k_1(z+d)}{\cosh 2k_1d}\sin 2(k_1x-\omega_1t)+\\&\frac{1}{2}{a_2}^2E_{12}^{\pm}\frac{\cosh 2k_2(z+d)}{\cosh 2k_2d}\times\\&\sin 2(k_2x-\omega_2t).\end{aligned} \tag{17}$$

Besides the surface condition, this special solution should also satisfy the Laplace governing equations and the bottom boundary condition:

$$\text{for } -d<z<0,\ \nabla^2\phi^{(2)}=0, \tag{18}$$

$$\text{for } z=-d,\ \frac{\partial\phi^{(2)}}{\partial z}=0. \tag{19}$$

Substituting Eq. (17) into Eqs (18) and (19), we have

$$E_{12}^{\pm}=\frac{G^{\pm}(\omega_1,\omega_2)}{D^{\pm}(\omega_1,\omega_2)}\ E_{11}^{+}=\frac{G^{+}(\omega_1,\omega_1)}{D^{+}(\omega_1,\omega_1)}\ E_{22}^{+}=\frac{G^{+}(\omega_2,\omega_2)}{D^{+}(\omega_2,\omega_2)}, \tag{20}$$

where $D^{\pm}(\omega_1,\omega_2)=gk_{\pm}\tanh(k_{\pm}d)-{\omega_{\pm}}^2$.

Utilizing the same principle, if an infinite number of wave components are considered, then the second order potential under the random wave condition can be obtained as follow:

$$\begin{aligned}\varphi^{(2)}(x,z,t)=&\sum_i\sum_{j>i}a_ia_j\frac{G^{\pm}(\omega_i,\omega_j)}{D^{\pm}(\omega_i,\omega_j)}\frac{\cosh k_{\pm}(z+d)}{\cosh k_{\pm}d}\times\\&\sin(k_{\pm}x-\omega_{\pm}t+\varepsilon_{\pm})+\frac{1}{2}\sum_i{a_i}^2\frac{G^{+}(\omega_i,\omega_i)}{D^{+}(\omega_i,\omega_i)}\times\\&\frac{\cosh 2k_i(z+d)}{\cosh 2k_id}\sin 2(k_ix-\omega_it+\varepsilon_i).\end{aligned} \tag{21}$$

For a random field of ocean waves, the surface displacement, velocity potential and Eulerian velocity to the second order are then given as

$$\eta^{(1)}(x,t)=\sum_i a_i\cos(k_ix-\omega_it+\varepsilon_i), \quad (22)$$

$$\phi^{(1)}(x,z,t)=\sum_i\frac{a_ig}{\omega_i}\frac{\cosh k_i(z+d)}{\cosh k_id}\times \sin(k_ix-\omega_it+\varepsilon_i), \quad (23)$$

$$\phi^{(2)}(x,z,t)=\sum_i\sum_{j>i}a_ia_j\frac{G^{\pm}(\omega_i,\omega_j)}{D^{\pm}(\omega_i,\omega_j)}\times \frac{\cosh k_{\pm}(z+d)}{\cosh k_{\pm}d}\sin(k_{\pm}x-\omega_{\pm}t+\varepsilon_{\pm})+ \frac{1}{2}\sum_i a_i{}^2\frac{G^{+}(\omega_i,\omega_i)}{D^{+}(\omega_i,\omega_i)}\frac{\cosh 2k_i(z+d)}{\cosh 2k_id}\times \sin 2(k_ix-\omega_it+\varepsilon_i), \quad (24)$$

$$u^{(1)}(x,z,t)=\frac{\partial\phi^{(1)}}{\partial x}=\sum_i a_i\omega_i\frac{\cosh k_i(z+d)}{\sinh k_id}\times \cos(k_ix-\omega_it+\varepsilon_i), \quad (25)$$

$$u^{(2)}(x,z,t)=\sum_i\sum_{j>i}a_ia_jk_{\pm}\frac{G^{\pm}(\omega_i,\omega_j)}{D^{\pm}(\omega_i,\omega_j)}\times \frac{\cosh k_{\pm}(z+d)}{\cosh k_{\pm}d}\cos(k_{\pm}x-\omega_{\pm}t+\varepsilon_{\pm})+ \sum_i a_i{}^2\frac{G^{+}(\omega_i,\omega_i)}{D^{+}(\omega_i,\omega_i)}\frac{\cosh 2k_i(z+d)}{\cosh 2k_id}\times k_i\cos 2(k_ix-\omega_it+\varepsilon_i), \quad (26)$$

$$x-x_0=\int_0^t u\mathrm{d}t\approx\sum_i -a_i\frac{\cosh k_i(z_0+d)}{\sinh k_id}\times \sin(k_ix_0-\omega_it+\varepsilon_i), \quad (27)$$

$$z-z_0=\int_0^t \omega\mathrm{d}t\cong\sum_i a\frac{\cosh k_i(z_0+d)}{\sinh k_id}\times \cos(k_ix_0-\omega_it+\varepsilon_i), \quad (28)$$

where $\omega_{\pm}=\omega_i\pm\omega_j$; $k_{\pm}=k_i\pm k_j$; $\varepsilon_{\pm}=\varepsilon_i\pm\varepsilon_j$; $D^{\pm}(\omega_i,\omega_j)=gk_{\pm}\tanh(k_{\pm}d)-\omega_{\pm}{}^2$; and

$$G^{\pm}(\omega_i,\omega_j)=-g^2\left[\frac{k_ik_j}{\omega_i\omega_j}\omega_{\pm}\times(1\mp\tanh k_id\tanh k_jd)+ \left(\frac{k_i{}^2}{2\omega_i\cosh^2 k_id}\pm\frac{k_j{}^2}{2\omega_j\cosh^2 k_jd}\right)\right]. \quad (29)$$

Again, the Lagrangian horizontal velocity can be derived from the Taylor's expansion:

$$u(x,z,t)=\sum_i c_i(a_ik_i)\frac{\cosh k_i(z_0+d)}{\sinh k_id}\times \cos(k_ix_0-\omega_it+\varepsilon_i)+ \sum_i\sum_{j>i}a_ia_jk_{\pm}\frac{G^{\pm}(\omega_i,\omega_j)}{D^{\pm}(\omega_i,\omega_j)}\times \frac{\cosh k_{\pm}(z_0+d)}{\cosh k_{\pm}d}\times \cos(k_{\pm}x_0-\omega_{\pm}t+\varepsilon_{\pm})+ \sum_i a_i{}^2\frac{G^{+}(\omega_i,\omega_i)}{D^{+}(\omega_i,\omega_i)}\frac{\cosh 2k_i(z_0+d)}{\cosh 2k_id}\times \sum_i\sum_j\frac{a_ia_j}{2}\frac{(k_i\omega_i+k_j\omega_j)}{\sinh k_id\cdot\sinh k_jd}\times [\cos(k_{-}x_0-\omega_{-}t+\varepsilon_{-})\cosh k_{+}(z_0+d)- \cos(k_{+}x_0-\omega_{+}t+\varepsilon_{+})\cosh k_{-}(z_0+d)]+ \sum_i c_i\frac{(a_ik_i)^2}{2\sinh^2 k_id}\cos 2(k_ix_0-\omega_it+\varepsilon_i)+ \sum_i c_i(k_ia_i)^2\frac{\cosh 2k_i(z_0+d)}{2\sinh^2 k_id}+O(ka)^3. \quad (30)$$

In this expression, the first term denotes the first harmonic, the third and fifth terms the second harmonic, and the second and fourth terms the interaction among the different wave components. The last term is the drift velocity in the random wave field, which is simply the algebraic summation of the drift velocity from all the wave components, i.e.,

$$u_{\rm s}=\sum_{i=1}^{\infty}c_i(k_ia_i)^2\frac{\cosh 2k_i(z_0+d)}{2\sinh^2 k_id} =2\int_0^{\infty}s(\omega)\omega k\frac{\cosh 2k(z_0+d)}{2\sinh^2 kd}\mathrm{d}\omega. \quad (31)$$

In deep water condition, the surface drift at z_0=0 would be

$$u_{\rm s}=2\int_0^{\infty}s(\omega)\omega k\,\mathrm{d}\omega, \quad (32)$$

which is identical to Eq. (1).

Equations (30) and (32) illustrate that when the wave energy spectrum is defined, the corresponding drift velocity would then be a time-independent quantity. In other words, the derivation implies an important outcome that the Stokes drift should not possess any fluctuation component, similar to the situation under the regular waves. Consequently, it should not cause any dispersion effect to the floating substances under the random wave field, which is contrary to the suggestion by Herterich and Hasselmann (1982) and others.

3 Experimental results and discussion

In the above section, the derivations have shown that there should not be a dispersion effect to the floating substances due to the surface drift in random waves. To verify this conclusion, we carried out an experimental study as described in the following.

3.1 *Experimental setup*

Experiments were conducted in a wave flume measuring 45.00 m long, 1.55 m wide and 1.50 m height (Fig. 1). The large flume size allowed deep-water waves to be generated. One end of the flume housed a two-dimensional piston type random wavemaker. The wave piston was controlled directly using the DHI active wave absorption control system (AWACS). Meanwhile, DHI wave synthesizer software was used to specify the types of wave required as well as their respective parameters. At the other end of the wave flume, an artificial absorbent beach was built to provide an efficient dissipation of the wave energy.

Four sensitive capacitance wave probes were mounted on a steel frame positioned at a range of 1.2–2.2 m from the wave paddle. The wave probes were capable of measuring wave heights to the nearest 0.5 mm. The probes had a small diameter

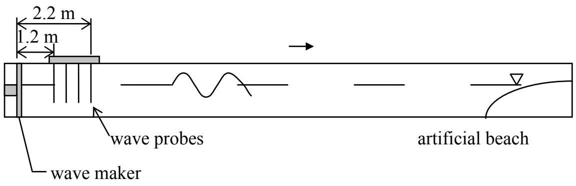

Fig. 1. Schematic layout of the wave flume (profile view).

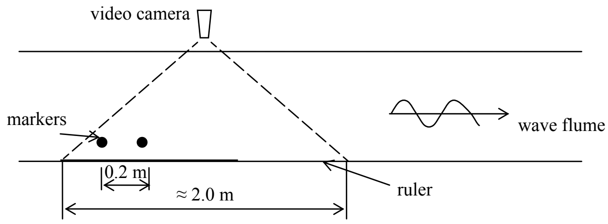

Fig. 2. Schematic layout of experimental set-up (plan view).

(approximately 0.6 cm) such that their placement would not result in significant modifications of the wave profile.

Small vinyl marks with a radius of about 1 cm were used as floating particles in the experiments. The vinyl marks had a density of 0.9 g/cm^3 and they were used to simulate the surface contaminant particles. They floated on the water surface without any relative movement to the surface water particles. A ruler is fixed on the wall of one side of the wave flume to determine the position of the markers. On the opposite side, a digital video camera mounted on a tripod was positioned beside the measurement area to capture the images of the particles' movement. The field of view of the camera covered a span of about 2 m, in which the position of the small vinyl marks and their coordinates in the images can be identified. The schematic layout of the experimental set-up is shown in Fig. 2.

3.2 *Experimental procedures*

Two vinyl marks were placed on the water surface along the wave flume. According to Eq. (27), the first order horizontal displacement should be of the same order as the wave height (maximum H_s=0.05 m in this study). The initial distance for the two marks was thus set to be 0.2 m to avoid their collision. An experiment was initiated only after the water was sufficiently still. This can be judged by visually inspecting the motion of the marks for 1 min or 2 min. Any discernible movement would indicate the presence of a residual current. After it was ensured that there were no residual currents, the digital video camera was switched on and the wave generator was activated.

The experimental duration was chosen to be less than 100 s to avoid the effects by the reflected waves. Prior to the study, we had examined the effectiveness of the artificial beach installed in terms of dissipating the incoming waves. It was found that the beach was quite effective and the reflected wave height was less than 10% within the range of the wave conditions tested. Given the wave period of 1.0 s, the group velocity of the wave train would be equal to 0.78 m/s. The distance between the test span and the end of the flume was approximately 30 m. Hence, it would take about 80 s for the wave train to cover the distance to-and-fro. With the experimental duration less than 100 s, the reflected waves were thus not expected to have a significant effect.

3.3 *Experimental results*

In the experiments, the Pierson-Moscowitz (P-M) spectrum was used to generate the random waves. Three different tests were conducted with H_s =0.03, 0.04 and 0.05 m respectively with the water depth d=0.8 m. The measured wave spectra, plotted in Fig. 3, show that the spectrum generated in the tank agrees well with the theoretical P-M spectrum. In order to verify the repetition of experiments, each of the three tests were repeated three times, and similar results were obtained.

The distances ΔL between the marker particles determined by the coordinate-difference in the video recordings at different time is shown in Fig. 4. It can be observed that they only oscillate within a small range (±0.05 m) in these experiments during the test duration. These experimental observations are distinctly different from those of Buick et al. (2001) who reported that the relative distance of two marker particles increases with time. In their measurements, Buick et al. (2001) state that the turbulent diffusion have a significant effect, which implies that the wave in their experiments does not match potential flow anymore. However, the wave generated in the current study meets the linear wave criteria. This can be the reason leading to the difference.

The comparison between the experimental observations and Herterich and Hasselmann's (1982) predictions is also presented in Fig. 4. Following Herterich and Hasselmann's (1982) theory, after 100 s, the relative distance between the two markers would have increased by 8, 10 and 12 cm corresponding to the significant wave height 0.03, 0.04 and 0.05 m, respectively (see Appendix A). Clearly, this predicted increase was not observed in our experiments. The experimental results thus reinforce the analysis that the stochastic Stokes drift in the random waves does not lead to the surface dispersion of floating substances.

4 Conclusions

The derivations of the Lagrangian velocities in both regular and random waves performed in this study illustrate that the Stokes drift velocity is a time-independent quantity that is among the components of the Lagrangian velocities, and thus it can also be considered as an instantaneous value. In this regards, the drift velocity should not possess any fluctuation once a wave spec-

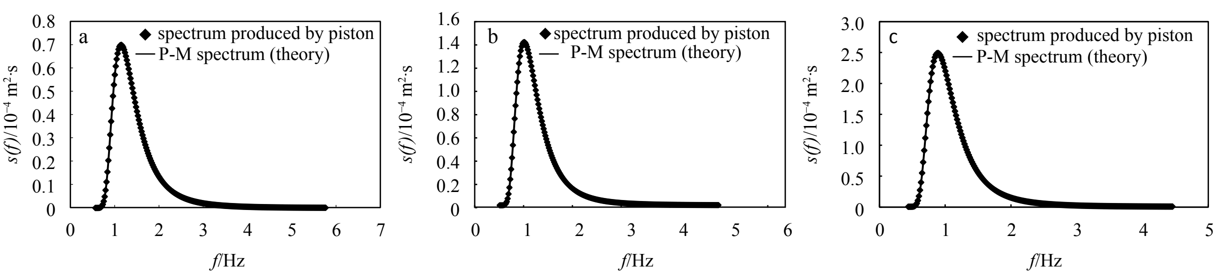

Fig. 3. Measured wave spectrum (P-M). a. H_s=0.03 m, b. H_s=0.04 m and c. H_s=0.05 m.

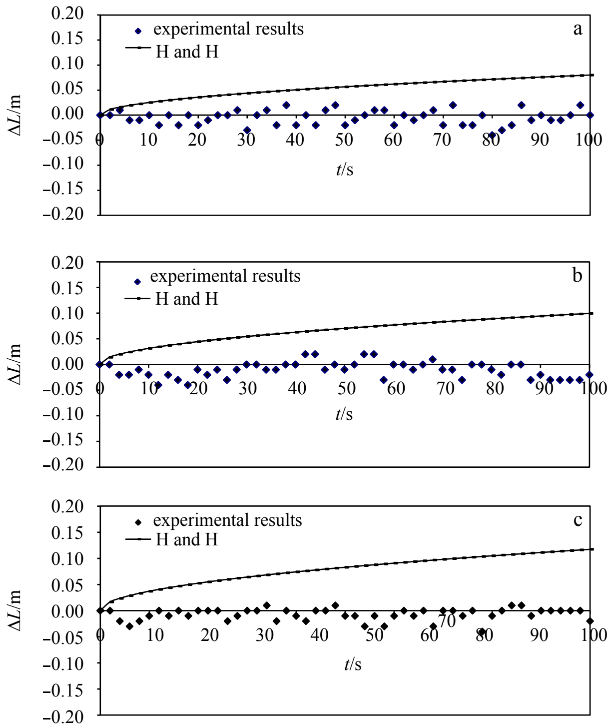

Fig. 4. The comparison of the changes in the relative distance the comparison of the results between the two markers, ΔL, between the experimental measurements and Herterich and Hasselmann's (1982) theory. a. H_s=0.03 m, b. H_s=0.04 m and c. H_s=0.05 m.

trum is defined. Since the fluctuation is a necessary mechanism for the dispersion effect predicted by Herterich and Hasselmann (1982), the existence of the Stokes drift in the random waves should therefore not be able to induce a dispersion effect on surface floating substances. The experimental findings also verify this conclusion.

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Appendix: The surface dispersion coefficient under the random waves with the P-M spectrum using Herterich and Hasselmann (1982) theory

The dispersion coefficient in the 3-D random wave condition formulated by Herterich and Hasselmann (1982) has been shown in Eq. (3). When the waves only travel in one direction such as in a wave flume, the spreading functions $G(\theta_1)$ and $G(\theta_2)$ will become δ-function:

$$G(\theta_1) = \begin{cases} \infty & \theta_1 = 0, \\ 0 & \theta_1 \neq 0, \end{cases} \quad \int_{-\pi}^{\pi} G(\theta_1)\,\mathrm{d}\,\theta_1 = 1, \tag{A1}$$

$$G(\theta_2) = \begin{cases} \infty & \theta_2 = 0, \\ 0 & \theta_2 \neq 0, \end{cases} \quad \int_{-\pi}^{\pi} G(\theta_2)\,\mathrm{d}\,\theta_2 = 1. \tag{A2}$$

Substituting Eqs (A1) and (A2) into the wave direction terms in Eq. (3), the dispersion coefficient on 2-D condition can then be obtained as

$$D_{xx} = \frac{4\pi}{g^2}\int_0^{\infty} \omega^6 s^2(\omega)\,\mathrm{d}\,\omega. \tag{A3}$$

Substituting the P-M spectrum, i.e., $s(\omega) = \frac{0.78}{\omega^5}\exp\left(-\frac{3.11}{\omega^4 {H_s}^2}\right)$ into the above equation:

$$D_{xx} = \frac{4\pi}{g^2}\int_0^{\infty} 0.78^2\omega^{-4}\exp\left(-\frac{6.22}{\omega^4 {H_s}^2}\right)\mathrm{d}\omega. \tag{A4}$$

Given the value of the significant wave height H_s, the dispersion coefficient can then be solved numerically by Eq. (A4). The change of the relative distance between the two markers, ΔL with time t can then be computed with the significant wave height for the three tests.