Effect of Stokes drift on upper ocean mixing

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Abstract
Stokes drift is the main source of vertical vorticity in the ocean mixed layer. In the ways of Coriolis – Stokes forcing and Langmuir circulations, Stokes drift can substantially affect the whole mixed layer. A modified Mellor-Yamada 2.5 level turbulence closure model is used to parameterize its effect on upper ocean mixing conventionally. Results show that comparing surface heating with wave breaking, Stokes drift plays the most important role in the entire ocean mixed layer, especially in the subsurface layer. As expected, Stokes drift elevates both the dissipation rate and the turbulence energy in the upper ocean mixing. Also, influence of the surface heating, wave breaking and wind speed on Stokes drift is investigated respectively. Research shows that it is significant and important to assessing the Stokes drift into ocean mixed layer studying. The laboratory observations are supporting numerical experiments quantitatively.

Key words: Stokes drift, Langmuir circulations, Coriolis – Stokes forcing, upper ocean mixing, Mellor-Yamada 2.5 turbulence model, wave breaking, surface heating

1 Introduction

In the ocean mixed layer, Stokes drift is the main source of vertical vorticity in the ways of Coriolis-Stokes forcing and Langmuir circulations (Poltonet al., 2005; Lewis and Belcher, 2004), as sketched in Fig. 1. For the former, the interactions of the Stokes drift with planetary vorticity altering the balance of the mean flow in the mixed layer. It deforms the planetary vorticity via a modified Coriolis force term in the linear horizontal momentum equations, which is called Coriolis – Stokes forcing (Hasselmann, 1970; Poltonet al., 2005). For the latter, Stokes drift associated with the mixed layer turbulence deforms the vorticity and generates Langmuir circulations (Craik and Leibovich, 1976; Leibovich, 1983, 1977; Teixeira and Belcher, 2002), which is believed to enhance the turbulent kinetic energy and regulate the depth of the mixed layer (McWilliams et al., 1997; Skyllingstad and Denbo, 1995).

Hasselmann (1970) first yielded Coriolis – Stokes forcing on the Eulerian momentum balance
when studying the interaction between the planetary vorticity and the Stokes drift. Madsen (1978) and Huang (1979) showed that the Coriolis – Stokes forcing changing the usual Ekman balance in the wind-driven mixed layer and the current profiles. But this work is only considered its effect on shallow depth. Jenkins (1987, 1986) developed the theory for more sophisticated presentations of the turbulent stress, Xu and Bowen (1994) developed for finite depths. The effect of this Coriolis – Stokes forcing on vertical profiles of the mean current is studied by Poltonet al. (2005) in the wind-driven ocean mixed layer. They found that although the Coriolis – Stokes forcing penetrates only a small fraction of the depth of the wind-driven layer, the current profile is substantially changed through the whole depth of that layer. Their work provides an evidence that the Coriolis – Stokes forcing is an important mechanism in controlling the dynamics of the upper ocean. However, in their work the wind and waves have been specified separately.

The prevailing theory of Langmuir circulations was proposed based on the interaction of Stokes drift with wind-driven surface shear current (Craik and Leibovich, 1976). Faller and Caponi (1978) called this prevailing theory the CL1 (tested by Faller, 1978; Mizuno, 1985 and others), and called another mechanism which was suggested by Craik (1977) and explored by Leibovich (1977) the CL2 (experiment studied by Faller and Caponi, 1978; Mizuno and Cheng, 1992; Nepf et al., 1995). Pollard (1977) provides a valuable summary of observation and an assessment of the theories existed in 1976. Leibovich (1983) reviewed the progress on Langmuir circulations, and since then, there had been substantial developments in both modeling and observations. Through numerical modeling (Maurel et al., 1997; Gnanadesikan and Weller, 1995; Dartus et al., 2000) and observations both in field (Agrawal et al., 1992; Drennan et al., 1992) and in laboratory measurements (Mizuno and Cheng, 1992; Araujo et al., 2001), the interests of researchers for Langmuir circulations were changed from explaining how Langmuir circulations are produced to how they affect upper ocean mixing. New methods of observation, notably side-scan Doppler sonar (Smith et al., 1987) and freely drifting instruments (Farmer and Li, 1995), or automated underwater vehicles (AUVs) (Thorpe et al., 2003) are developed. A large eddy simulation (LES henceforth) model (Skyllingstad and Denbo, 1995; McWilliams et al., 1997) is popular utilized in a Langmuir circulation study. In recent years, modeling of Langmuir circulations is still focus on LES approach (Noh Y et al., 2006) and new methods of observations are popular as well (Thorpe et al., 2003). However, Langmuir circulation interaction with other mechanism, like wave

![Fig. 1. Schematic illustrating the orbital path for a particle under a wave is tilted by Coriolis – Stokes forcing: \( f \times u_s \) (Hasselmann, 1970) (a) and sketch of Langmuir circulations from the lecture notes of William Emery (http://cscr.colorado.edu/asen5215/) (b).]
breaking and internal waves (Chini and Leibovich, 2003), seems a new trend, and not much is known about their interrelations.

The upper ocean mixing plays an important role in the exchange of energy, momentum, heat and matter between the atmosphere and the ocean. Understanding Stokes drift's mechanism is essential to clear the patterns of mixed layer. Dynamics like wave breaking and surface heating in the mixed layer govern the sea-surface fluxes of heat and gases. Especially, the vertical velocity of Langmuir circulations below the convergence region (downwelling zone) can be as high as a few tens which can lead to bubble entrainment and transport to greater depths. Also, the Coriolis – Stokes forcing changed the current profile substantially through the whole depth of the wind-driven mixed layer. Thus, except the wave breaking and the surface heating, Stokes drift is undoubtedly important to upper ocean mixing.

Though research on Stokes drift made a great development, the influence of surface heating and wave breaking together with Stokes drift on the upper ocean mixing is not very clear until now. Therefore, in order to understand the role of Stokes drift in the upper ocean mixing, this article uses a modified Mellor – Yamada 2.5 level turbulence closure model (MY2.5 henceforth) to parameterize Stokes drift' effects conventionally. It is seems that only Kantha and Clayson (2004) tried from this aspect till now. They attempt to parameterize the effect of wave-breaking and Stokes drift on turbulence in the upper ocean mixing, using the Kantha and Clayson (1994) two-equation second moment closure model. However, differing from them, this article is concentrating on the injection of Stokes drift into the basic MY2.5 and focuses on the role of Stokes drift compared with wave-breaking and surface heating (cooling) respectively in the upper ocean, and emphasize the importance of Stokes drift more clearly. The theory of the MY2.5 and modified model about Stokes drift are described in Section 2. Compared with heating and wave breaking in the ocean mixed layer, the important role of Stokes drift will be illuminated through numerical simulation in Section 3. Furthermore, the influences of the surface heating, wave breaking and wind speed on Langmuir circulation are investigated respectively in Section 4. In Section 5, the laboratory data from Mizuno and Cheng (1992) are used to test our work and the result is well consistent. In the end, further discusses are performed and conclusions are made in Section 6.

2 Stokes drift with modified MY2.5

2.1 Model of MY2.5

The numerical model used in this paper is based on the MY2.5 level model (Mellor and Yamada, 1982) which neglects material derivative and diffusion terms subsequent to the Mellor and Yamada (1974). The basic equations are as follows:

$$\frac{\partial U}{\partial t} - fV = \frac{\partial}{\partial z} \left( K_v \frac{\partial U}{\partial z} \right),$$  \hspace{1cm} (1)

$$\frac{\partial V}{\partial t} + fU = \frac{\partial}{\partial z} \left( K_v \frac{\partial V}{\partial z} \right),$$  \hspace{1cm} (2)

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left( K_h \frac{\partial T}{\partial z} \right) - \frac{\partial R}{\partial z},$$  \hspace{1cm} (3)

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial z} \left( K_h \frac{\partial S}{\partial z} \right),$$  \hspace{1cm} (4)

where the horizontal coordinate axes $x$ and $y$ are fixed on the undisturbed surface, the vertical axis $z$ is positive upward, $U$ and $V$ are the mean velocity in $x$ and $y$ directions respectively, $\bar{U} = (U, V)$; $T$ is temperature; $S$ is the salinity; $t$ is time. $f = 2 \omega \sin \varphi$, $\omega$ is the earth rotating angular velocity, $\varphi$ is the latitude; $K_v$ and $K_h$ are vertical diffusion parameters with $K_v = q l S_m$ and $K_h = q l S_n$, $S_m$ and $S_n$ are functions of Richardson number, $q = \sqrt{u^2 + v^2}$, is the turbulence velocity scale and $l$ is the length scale. The turbulence velocity scale and the length scale can be
derived through a conservation equation for turbulence kinematics equation for the turbulence velocity scale to the power 2 and mixed length equation for the long scale:

\[
\frac{\partial^2 U}{\partial t} = \frac{\partial}{\partial z} \left[ K_u \frac{\partial^2 U}{\partial z^2} \right] + 2K_m \left[ \left( \frac{\partial U}{\partial z} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2 \right] + 2K_b N^2 - 2 \frac{q^3}{B_l l} + F_s, \quad (5)
\]

\[
\frac{\partial^2 V}{\partial t} = \frac{\partial}{\partial z} \left[ K_u \frac{\partial^2 V}{\partial z^2} \right] + E_i l \left[ K_m \left( \frac{\partial U}{\partial z} \right)^2 + \frac{\partial V}{\partial z} \right] + 2K_b N^2 - 2 \frac{q^3}{B_l l} + F_{vi}. \quad (6)
\]

The first term on the right of (5) is vertical diffusion, the second denotes shear production, and the other three are buoyancy term, diffusion term and external forces. \( \vec{W} \) is wall proximity function with \( \vec{W} = 1 + E_s (U/L)^2 \), \( L \) is the distance from \( z \) to sea surface. The boundary condition on \( z = 0 \)

\[
\rho_s K_m \left( \frac{\partial U}{\partial z} \right)_z = (\tau_s, \tau_s), \quad (7)
\]

\[
\rho_s c_p K_m \left( \frac{\partial T}{\partial z} \right)_z = (Q_{nt}, Q_s), \quad (8)
\]

\[
q^2 = B_l^2 u_s^2, \quad (9)
\]

\[
q^2 l = 0; \quad (10)
\]

\[
z = -H
\]

\[
\rho_s K_m \left( \frac{\partial U}{\partial z} \right)_z = (\tau_s^b, \tau_s^b), \quad (11)
\]

\[
\rho_s K_b \left( \frac{\partial T}{\partial z} \right)_z = 0, \quad (12)
\]

\[
q^2 = B_l^2 u_{s,b}^2, \quad (13)
\]

\[
q^2 l = 0, \quad (14)
\]

where \( \rho_s \) is density of water; \( \tau = (\tau_s, \tau_s) \) and \( \tau^b = (\tau_s^b, \tau_s^b) \) are the stress vector on surface and bottom; \( Q_{nt} \) and \( Q_s \) are net heat flux and sensitive heat flux; \( u_s \) and \( u_{s,b} \) are the friction velocities on sea surface and bottom.

2.2 Modified model considering Stokes drift

Hasselmann (1970) found that the interaction between the planetary vorticity and the Stokes drift yields a force on the Eulerian momentum balance: \( f \times u_s \). According to Polton et al. (2005) this process is referred to as the Coriolis-Stokes forcing, and the momentum equations (1) and (2) are changed as Eqs (15) and (16). According Craik and Leibovich (1976), LCSs are formed due to the interaction of the wind-driven surface shear with the Stokes drift of the surface waves. Then turbulence kinematic equation for the turbulence velocity scale to the power 2 [Eq. (5)] and mixed length equation for the length scale [Eq. (5) and (6)] are modified by the appearance of a vortex force term: \( u_s \times (\nabla \times \vec{U}) \). Details see McWilliams et al. (1997).

\[
\frac{\partial U}{\partial t} - f(V + V_s) = \frac{\partial}{\partial z} \left[ K_m \frac{\partial U}{\partial z} \right], \quad (15)
\]

\[
\frac{\partial V}{\partial t} + f(U + U_s) = \frac{\partial}{\partial z} \left[ K_m \frac{\partial V}{\partial z} \right]. \quad (16)
\]

Similarly, the turbulent energy equation changed to follows as those of Kantha and Clayson (2004):

\[
\frac{\partial q^2}{\partial t} = 2K_m \left[ \left( \frac{\partial U}{\partial z} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2 + \left( \frac{\partial U}{\partial z} \right) \left( \frac{\partial U}{\partial z} \right) \right] + \frac{\partial}{\partial z} \left[ K_s \frac{\partial^2 U}{\partial z^2} \right] + 2K_b N^2 - 2 \frac{q^3}{B_l l} + F_s, \quad (17)
\]

\[
\frac{\partial q^2 l}{\partial t} = E_i l K_m \left[ \left( \frac{\partial U}{\partial z} \right)^2 + \left( \frac{\partial V}{\partial z} \right)^2 + \left( \frac{\partial U}{\partial z} \right) \left( \frac{\partial U}{\partial z} \right) \right] + \frac{\partial}{\partial z} \left[ K_s \frac{\partial^2 U}{\partial z^2} \right] + E_i l K_s N^2 - 2 \frac{q^3}{B_l l} + F_{vi}. \quad (18)
\]

The Stokes drift velocity appeared in the above equations is estimated by PM spectra (Pierson and Moskowitz, 1964) as obtained by McWilliams and Restrepo (1999):

\[
U_s(z) = 0.04U_{10} \exp \left[ -\frac{4\sqrt{g} l}{U_{10}} \left| z \right| \right], \quad (19)
\]

where \( U_{10} \) is the wind speed at 10 m above the mean sea surface; and \( V_s \) equaling for our experiments is only one-dimension. The other main parameters are listed in Table 1 (Sun et al., 2005).
Table 1. Empirical model parameters

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<th>B_1</th>
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<th>E_1</th>
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2.3 Wave breaking and surface heating

According to Kraus and Turner (1967), the surface turbulence energy due to wave breaking was parameterized with

\[- \left[ \frac{P}{\rho} + \frac{1}{2} u_*^2 \right] w(0) = m u_*^3, \]  

(20)

where \( m \) is a constant; and \( u_* \) is the friction velocity on the sea surface. Sun et al. (2005) discussed the effects of wave breaking on the turbulence kinetic energy by using Eq. (21) instead of Eq. (9):

\[ K_t \frac{\partial^2 \theta}{\partial z^2} = m u_*^3, \]  

(21)

where the parameter \( m \) is a wave energy factor and can be taken as a constant value of 100 in the calculations according to Craig and Banner (1994).

When surface heating is considered, different values of the net heat flux (it being greater than 0 means heating and being less than 0 cooling) are used in Eq. (8).

3 Model results compared with wave breaking and surface heating

In order to compare the effects of Stokes drift, surface heating and wave breaking in upper ocean mixing, six experiments are designed under the same wind field with and without Stokes drift:

Exp. 1, without wave breaking and surface heating, just compares eddy viscosity \( (K_e) \) calculated by MY2.5 with that of modified MY2.5 which assess the effect of Stokes drift. In Fig. 2a, the dotted line denotes the result of MY2.5 without Stokes drift. This line also appeared in Figs b – f for comparing with other cases.

![Fig. 2](image-url)

Fig. 2. Eddy viscosity with (the solid line) and without (the dash line) Stokes drift. The dotted lines in each subfigure are calculated by MY2.5 model. a. calculated by MY2.5, b. with wave breaking, c. with surface heating, d. with surface cooling, e. with surface heating and wave breaking and f. cooling and wave breaking. Wind speed is 15 m/s in each experiment.
Exp. 2, after wave breaking is injected into the model, the eddy viscosity is calculated for cases with and without Stokes drift, and the corresponding results are shown in Fig. 2b.

Exp. 3, considering surface heating with the net heat flux equaling 85 W/s², the effects on the eddy viscosity with and without Stokes drift are shown in Fig. 2c.

Exp. 4, considering surface cooling with the net heat flux equaling -75 W/s², the effect on the eddy viscosity with and without Stokes drift are shown in Fig. 2d.

Exp. 5, considering both wave breaking and surface heating, the synthetically effect on the eddy viscosity with and without Stokes drift are shown in Fig. 2e.

Exp. 6, considering both wave breaking and surface cooling, the synthetic effect on the eddy viscosity with and without Stokes drift are shown in Fig. 2f.

Figure 2 illustrates the eddy viscosity changed obviously with effect of Stokes drift. It is known that surface heating weakens mixing process mainly because it enhances the stratification of the ocean while cooling strengthens the mixing. Stokes drift enhances this feature further when their effects are included as shown in Figs 2c and d. When the synthetic effect of wave breaking, surface heating and Stokes drift are considered, the trend in Figs 2c and d is agrgrandized as shown in the Figs 2e and f. But at the sea surface the vertical diffusion strengthens clearly due to wave breaking. From these six experiments, it is shown that wave breaking enhances the mixing mainly concentrated at the surface layer. In Fig. 2b the maximum of vertical diffusion parameter (Kv) is nearly 0.9 W/s², 16 times larger than that of the maximum calculated by MY2.5. However, it is Stokes drift that enhances the mixing on the entire field. The effect on the vertical diffusion parameter (Kv) of Stokes drift with wave breaking together is nearly twice as much as that of wave breaking only. Therefore, even though surface heating and wave breaking are essential in the upper ocean mixing, the Stokes drift is more important. In the next section, the influence of the different heating flux on the Stokes drift will be discussed in more detail. But in this section our focus is emphasized on the Stokes drift importance which compared with wave breaking and surface net heat flux. Figure 3 shows that the depth-integrated terms in turbulence kinetic budgets with Stokes drift and wave breaking are compared with those obtained when wave breaking are considered only.

In Fig. 3a, it is shows that the shear production is decreased by Stokes drift. It is mainly because that Stokes drift enhances the mixing, accordingly enhances the vertical velocity and slows down the mean velocity, therefore, decreases the shear production. In the mean time, the dissipation increases due to mixing intensified by Stokes drift as it shows in Fig. 3c. On the basis of the supposition of partial equilibrium by Craig and Banner (1994), the buoyancy force will increase also, just as it shown in Fig. 3b.

![Figure 3](image_url)

**Fig. 3.** Depth-integrated terms in turbulence kinetic budgets with Stokes drift and wave breaking (solid line) and with wave breaking only (dashed line). a. Shear production term. b. Buoyancy force term. c. Dissipation term. \( t_i \) is intertial period.
4 Influence of surface heating, wave breaking and wind on Stokes drift

In this part, the effect of wind and surface heating on the Stokes drift is analyzed from another aspect which is different from Section 3. From Figs 4a and b, it is shows that the eddy viscosity enhances more than five times as wind speed is only twice as fast as before. Figure 4b manifests contains both Stokesly drift and wave breaking. When they work together under the same condition, the eddy viscosity strengthens further more, increases from 0.218 with only Stokes drift to 0.512 with both effects at the peak. And the surface turbulence enhances due to the wave breaking, the eddy viscosity increases form 0.412 with only wave breaking to 0.763 with together. Wave breaking dominates the process and strengthens the mixing strongly in the surface while Stokes drift strengthens at greater depth. Furthermore, when wind speeds up, it can be seen that upper ocean mixing affected by Stokes drift is much stronger than by wave breaking. In other words, Stokes drift is much more sensitive to the wind.

![Fig. 4. Eddy viscosity with wind of 15 m/s (dash line) and wind of 30 m/s (solid line), a. Considering Stokes drift and b. considering effects of Stokes drift and wave breaking.](image)

![Fig. 5. Eddy viscosity with surface cooling (75 W/m², dash line and 300 W/m², solid line); (a) and surface heating (85 W/m², dash line and 340 W/m², solid line) (b).](image)

Figure 5 describes the Stokes drift response with different flux to surface heating or cooling. Figure 5a shows that, the maximum of eddy viscosity increases from 0.08 to 0.16 m²/s, when net surface heating
flux decreases from $-75$ to $-300$ W/m$^2$. Cooling strengthens mixing process mainly because cooling weakens the stratification of the ocean while heating decreases the mixing. Therefore, it can be seen that Stokes drift is sensitive to the cooling. When the surface heating flux increases from 85 to 340 W/m$^2$, the maximum of eddy viscosity decreases from 0.052 to 0.031 m$^2$/s as shown by Fig. 5 b.

5 Model test

Through the above analysis, it is no doubted that Stokes drift plays a key role in the upper ocean mixing process. However, there is still a gap between models and observations. To connect them, our model should be tested by some data. Fortunately, Mizuno and Cheng (1992) have done an experimental study on Stokes drift in a laboratory tank. In their wind-only experiment study, they succeeded in observing a pair of Stokes drift in a closed rectangular tank under the action of a uniform wind. The $v_i$ appeared in Eq. (3c) of Mizuno and Cheng (1992) is consistent with our eddy viscosity ($K_m$). Water depth in the tank is 1.2 m, and tank wide is 1.5 m. The horizontal coordinate axes $x$ and $y$ are fixed on the undisturbed surface, and axis $x$ along with the tank is located in the middle of it while $y$ axis across the tank, the vertical axis $z$ is positive upward. One group data is chosen from Table 3 of Mizuno and Cheng (1992) which is observed on one side of the sidewalls ($y = -0.875$). Because these data are located near the boundary of the tank where the turbulence is strong extraordinarily, some of them are very large accordingly. Besides, the Stokes drift is studied in an closure tank, the wind blowing must induce a large circulation, its velocities become large rapidly and orient to the wind direction at the surface while the reverse circumstances appear at the bottom. Thus, some normalized eddy viscosities are chosen to compare with the eddy viscosity as Stokes drift' effects are considered, as it shown in Fig. 6. The solution shown that the eddy viscosity calculated by the model with Stokes drift is well in agreement with the data.

![Graph](image_url)

Fig. 6. Eddy viscosity (solid line) plotted as considering Stokes drift' effect and compared with measurements data (•) by Mizuno and Cheng (1992).

6 Conclusions

On the basis of the MY2.5 level model, the effects of Stokes drift on upper ocean mixing are parameterized conventionally. Six experiments were tested for different mixing processes. Through comparing wave breaking with surface heating, Stokes drift is regarded as the most important mechanics in the whole upper ocean mixing as it is expected.

On the other hand, interactions of Stokes drift with wave breaking, heating and wind speed are tested respectively. Results show that: (1) comparing with wave breaking, Stokes drift are more sensitive to wind varying; (2) when wave breaking and Stokes drift exist at the same time they enhance the turbulence effect on upper ocean mixing each other which strengthens the mixing further; (3) Stokes drift is sensitive to wind and surface heating. The conclusion is that the Stokes drift is more active a the other processes. Certainly, one-dimension model has
its limit in many ways though it can describe the importance of Stokes drift in the upper ocean mixing. In this paper the wind and wave have been assumed to be the same direction. We will examine the cases when they have different directions in the future so as to compare our results with the observations in the real ocean.

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