Numerical study of pollutant movement in waves and wave-induced long-shore currents in surf zone

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Abstract

Water waves, wave-induced long-shore currents and movement of pollutants in waves and currents have been numerically studied based on the hyperbolic mild-slope equation, the shallow water equation, as well as the pollutant movement equation, and the numerical results have also been validated by experimental data. It is shown that the long-shore current velocity and wave set-up increase with the increasing incident wave amplitude and slope steepness of the shore plane; the wave set-up increases with the increasing incident wave period; and the pollutant movement proceeds more quickly with the increasing incident wave amplitude and slope steepness of the shore plane. In surf zones, the long-shore currents induced by the inclined incident waves have effectively affected the pollutant movement.

Key words: water waves, hyperbolic mild-slope equation, longshore currents, pollutants

1 Introduction

Shallow coastal zones are extremely dynamic regions due to the complicated topography, especially in mild-slope zones, where the hydrodynamics are more complicated as evident in the refraction, diffraction, collection and breaking of water waves, as well as the presence of breaking-wave-induced near-shore currents. As a result, the pollutants movements in coastal zones are also complicated due to the complicated hydrodynamics in shallow coastal zones. In these zones, a great deal of domestic and industrial sewage has been drained into seawater, and coastal environment has been seriously destroyed consequently. Hence, it is essential to analyze the pollutant movement in coastal hydrodynamic factors to maintain a healthy environment in coastal zones.

Several researchers have made much progress both on numerical and experimental modeling near-shore water waves, near-shore currents and pollutant movement in coastal zones (Bao et al., 2006; Tang et al., 2006; Tao and Han, 2003; Wang, 2001; Zheng et al., 2000; Katopodi and Ribberink, 1992; Fischer, 1988; Madsen and Larsen, 1987; Falconer, 1986; Van-Rijn, 1986; Berkhoff, 1972; Longuet-Higgins, 1970; ). However, due to its complicated hydrodynamics in shallow coastal zones, the hydrodynamics in these zones are still needed to be studied, and what's more pollutant movements in

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these zones have still less been studied. In this study, water waves and breaking-wave-induced longshore currents have been numerically simulated based on the hyperbolic mild-slope equation and nearshore current equation. The wave radiation stresses have been calculated based on the variables in hyperbolic mild-slope equation, and the wave-induced longshore currents have been numerically simulated based on these as well as the pollutant movement in waves and wave-induced long-shore currents has been numerically simulated based on these. The numerical results of water waves, wave-induced long-shore currents and pollutant movement in waves and currents have also been analyzed and validated by experimental data.

2 Numerical model

2.1 Hyperbolic mild-slope equation

The elliptic mild-slope equation put forward by Berkhoff (1972), which takes the reflection, refraction and diffraction of water wave effects into account, is an effective wave model for numerical simulating propagation of waves in coastal mild-slope zones. Several researchers have put forward parabolic mild-slope equation and hyperbolic mild-slope equation based on elliptic mild-slope equation for the lower efficiency in numerical simulating elliptic mild-slope equation due to its large-scale matrix calculation. The following equations put forward by Larsen and Madsen (1987) have been used as the governing equations for simulating propagating water waves:

\[
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} \left[ U(h + \eta) \right] + \frac{\partial}{\partial y} \left[ V(h + \eta) \right] = 0, \quad (5)
\]

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + g \frac{\partial \eta}{\partial x} + \frac{1}{\rho(h + \eta)} \times \left( \frac{\partial S_m}{\partial x} + \frac{\partial S_n}{\partial y} \right) - \frac{1}{\rho(h + \eta)} \times (\tau_m - \tau_m) - A_m = 0, \quad (6)
\]

\[
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + g \frac{\partial \eta}{\partial y} + \frac{1}{\rho(h + \eta)} \times \left( \frac{\partial S_m}{\partial x} + \frac{\partial S_n}{\partial y} \right) - \frac{1}{\rho(h + \eta)} \times (\tau_m - \tau_m) - A_m = 0, \quad (7)
\]

where \(i\) is imaginary unit; \(S\) is the complex wave evaluation; \(P\) and \(Q\) are the integral function of water particle velocity along water depth in \(x\) and \(y\) direction respectively; \(\omega\) is wave angular frequency; \(k\) is wave number; \(C = \frac{\omega}{k}\) is wave phase velocity; \(C_s = \frac{\partial \omega}{\partial k}\), is wave group velocity. In surf zone, the wave heights \(H\) are controlled by the following relation:

\[H = y_h,\]

where \(h\) is local water depth; \(\gamma = 0.6 - 0.8\), is the ratio parameter for guiding the breaking waves.

2.2 Wave-induced current model

The wave-induced currents may be governed by the following equations:

\[
\text{where } U \text{ and } V \text{ are the wave-induced current velocities in } x \text{ and } y \text{ directions, respectively; } \eta \text{ is the wave set-up or set-down; } S_m, S_n, S_r, \text{ and } S_s \text{ are the wave radiation stress components; } \tau_m \text{ and } \tau_r \text{ are the surface friction stresses in } x \text{ and } y \text{ directions, respectively; } A_m \text{ and } A_s \text{ are the lateral mixing stresses in } x \text{ and } y \text{ directions, respectively.}
\]

The wave radiation stresses are the mainly driving forces of wave-induced currents (Longuet-Higgins, 1970). It is difficult to ascertain the wave
propagating directions due to the reflection, refraction and diffraction of water waves, and for the airy waves, wave radiation stresses may be derived from the wave potential. The wave radiation stresses used here are calculated based on the variables in the hyperbolic mild-slope equation:

\[
S_w = \frac{\rho g}{4} \left( \int \frac{\partial S}{\partial x} \left( 1 + \frac{2kh}{\sinh 2kh} \right) + \left( \int \frac{\partial S}{\partial y} \left( 1 + \frac{2kh}{\tanh 2kh} \right) \right) \right) \times \left( \int \frac{\partial S}{\partial x} \left( 1 + \frac{2kh}{\sinh 2kh} \right) \right),
\]

\[
S_n = \frac{\rho g}{4} \left( \int \frac{\partial S}{\partial y} \left( 1 + \frac{2kh}{\sinh 2kh} \right) + \left( \int \frac{\partial S}{\partial x} \left( 1 + \frac{2kh}{\tanh 2kh} \right) \right) \right) \times \left( \int \frac{\partial S}{\partial y} \left( 1 + \frac{2kh}{\sinh 2kh} \right) \right),
\]

\[
S_{ny} = S_{ny} = \frac{\rho g}{4} \left( \Re \left( \frac{\partial S}{\partial x} \frac{\partial S^*}{\partial y} \right) \right) \times \left( \int \frac{\partial S}{\partial y} \left( 1 + \frac{2kh}{\sinh 2kh} \right) \right),
\]

where \( S^* \) is the conjugate complex of \( S \). Much progress on the bottom friction stress in waves and currents has been made by several researchers. However, the conclusions on the mechanism of it are still not identical due to the complicated hydrodynamics in waves and currents. For the weak currents or the currents perpendicular to the waves, the bottom friction stresses may be governed by the following expressions (Longuet-Higgins, 1970):

\[
\tau_{bx} = \frac{4}{\pi} \rho c_i u_w U,
\]

\[
\tau_{by} = \frac{2}{\pi} \rho c_i u_w V,
\]

where \( u_w = 2\pi a_0/T \), is the amplitude of the bottom wave particle velocity, and \( a_0 = H/(2 \sinh kh) \), is the amplitude of the bottom wave particle movement; \( c_i \) is the bottom friction coefficient in waves and currents.

The lateral mixing stresses are governed by the following expressions:

\[
A_{wx} = \frac{\partial}{\partial x} \left( \mu \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial U}{\partial y} \right),
\]

\[
A_{wy} = \frac{\partial}{\partial x} \left( \mu \frac{\partial V}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial V}{\partial y} \right),
\]

where \( \mu \) is the lateral mixing coefficient governed by the formula:

\[
\mu = N x_1 \sqrt{gh},
\]

where \( x_1 \) is the distance between the wave breaking point to the shore line; \( N \approx 0.016 \), is a non-dimensional coefficient.

In this study, the following boundary conditions for the wave-induced currents are specified. At the offshore wave incident boundary, the mean water level is specified, and no-flow boundary condition is used for the wave-induced currents:

\[
\eta = 0,
\]

\[
U = 0,
\]

\[
V = 0,
\]

At the other boundaries, the continual boundary condition is adopted:

\[
\frac{\partial \eta}{\partial \hat{n}} = 0,
\]

\[
\frac{\partial U}{\partial \hat{n}} = 0,
\]

\[
\frac{\partial V}{\partial \hat{n}} = 0,
\]

where \( \hat{n} \) is the normal direction of the boundary.

2.3 Model for pollutant movement in waves and currents

The model for pollutant movement in waves and currents is guided by the formula:

\[
\frac{\partial c}{\partial t} + U \frac{\partial c}{\partial x} + V \frac{\partial c}{\partial y} = \frac{1}{h + \eta} \left( \frac{\partial}{\partial x} \left( h + \eta \right) D_{ww} \frac{\partial c}{\partial x} \right) + \frac{\partial}{\partial y} \left( h + \eta \right) D_{ww} \frac{\partial c}{\partial y} + S_w,
\]

where \( c \) is the pollutant concentration averaged in water depth; \( D_{ww} \) and \( D_{ww} \) are the diffusion coeffi-
coefficients of pollutant in waves and currents in $x$ and $y$ directions respectively; $S_m$ is the pollutant source terms.

The expression for the pollutant diffusion coefficient in waves and currents is as follows (Katopodi and Ribberink, 1992):

$$D_w = D_e + D_w', \quad (23)$$

where $D_e$ is the diffusion coefficient of pollutants in currents controlled by the formula (Falconer, 1986):

$$D_e = \frac{(k_1 U^2 + k_2 V^2) h \sqrt{g}}{Ch \sqrt{U^2 + V^2}}, \quad (24)$$

$$D_{w'} = \frac{(k_1 V^2 + k_2 U^2) h \sqrt{g}}{Ch \sqrt{U^2 + V^2}}, \quad (25)$$

where $D_w$ and $D_{w'}$ are the components of the diffusion coefficient of pollutants in currents in $x$ and $y$ directions respectively; $Ch$ is the Chezy coefficient; $k_1$ and $k_2$ are non-dimensional parameters, and have values of 5.93 and 0.15 respectively. The diffusion coefficients $D_w$ and $D_{w'}$ of pollutant in waves in $x$ and $y$ directions are controlled by the formula (Van- Rijn, 1986):

$$D_w = D_{w'} = 0.035 \alpha \frac{hH}{T}, \quad (26)$$

where $\alpha$ is the empirical factor with different values in different zones, and the following expression is used:

$$\alpha = \begin{cases} 
5.5H/k - 2.0 & \text{for surf zone}, \\
1.0 & \text{outer surf zone}. 
\end{cases} \quad (27)$$

Before the pollutant transport model can be solved numerically, initial and boundary conditions must be specified. In this study, the initial value for the pollutant concentration is set according to the pollutant initial state to be zero. If the pollutant concentration at the boundaries is known, the boundary conditions for the pollutant transport model can be set easily. However, it is usually hard to be directly measured and the studied pollutant concentration field is usually finite in space. Hence, proper boundary conditions should be set. It is usually assumed that the pollutant concentration at the boundaries is very small compared with its central concentration, and the pollutant boundary conditions are set as follows: at the boundary the current flows into the computation field, the pollutant concentration is set as its value far off the computation field, which is

$$c = 0. \quad (28)$$

At the boundary the pollutant flows out of the computation field, it is assumed that the pollutant concentration is uniform, and the boundary conditions, if the diffusion effect of the pollutant at the boundary is neglected, are set as follow:

$$\frac{\partial c}{\partial t} + \vec{V} \cdot \nabla c = 0. \quad (29)$$

The finite difference method is used to solve the numerical models in the present study.

3 Validation of the numerical model

The experimental study of pollutant movement in waves and wave-induced long-shore currents was conducted at the State Key Laboratory of Coastal and Offshore Engineering in Dalian University of Technology. The experimental topography set is shown in Fig. 1, where a mild slope plane is set inclined to the wave generator and long shore currents are induced as water waves propagate and break in the plane. The parameters for the four experimental cases are listed in Table 1, where $h_0$ is the still water depth before the plane; $\theta$ is the angle between the plane and incident waves; $H_0$ is the incident wave height; $L$ is the pollutant drainage site from the shore line, and the pollutant was released as the continual source style. The numerical models run on the numerical domain on which the coordinate is set parallel to the plane slope and the $x$ direction is set along the offshore direction and $y$ direction is set parallel to longshore direction. The wave-induced current models are run until they reach approximately a steady state.
Table 1. Experimental and numerical model parameters for pollutant movement in waves and wave-induced long-shore currents

<table>
<thead>
<tr>
<th>Case</th>
<th>Plane slope</th>
<th>$h_0/m$</th>
<th>$\theta/^{\circ}$</th>
<th>$H_0/m$</th>
<th>$T/s$</th>
<th>$\gamma$</th>
<th>$c_i$</th>
<th>$N$</th>
<th>$L/m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1:40</td>
<td>0.45</td>
<td>30</td>
<td>0.05</td>
<td>1.0</td>
<td>0.7</td>
<td>0.009</td>
<td>0.001 2</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
<td>1:40</td>
<td>0.45</td>
<td>30</td>
<td>0.09</td>
<td>1.0</td>
<td>0.7</td>
<td>0.009</td>
<td>0.001 2</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>1:40</td>
<td>0.45</td>
<td>30</td>
<td>0.05</td>
<td>2.0</td>
<td>0.7</td>
<td>0.009</td>
<td>0.001 2</td>
<td>3.0</td>
</tr>
<tr>
<td>4</td>
<td>1:100</td>
<td>0.18</td>
<td>30</td>
<td>0.05</td>
<td>2.0</td>
<td>0.6</td>
<td>0.0065</td>
<td>0.001 2</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Fig. 1. Experimental topography set of movement of pollutants in waves and wave-induced longshore currents.

The comparisons between the experimental (Wang, 2001) and numerical results of water wave heights, wave set-up and wave-induced longshore current velocities are shown in Figs 2 ~ 5, in which the $x$ direction is set along the offshore direction. It can be seen from Figs 2 ~ 5 that the numerical results are in good accordance with the experimental data in the whole for all four cases. It is found in Cases 1 and 2 that the wave-induced longshore current velocities and wave set-up increase with the increasing of incident wave amplitudes; as well as it is shown in Cases 1 and 3 that the wave set-up increases with the increasing incident wave periods; it is also shown in Cases 3 and 4 that the wave set-up and wave-induced long-shore current velocities increase with the increasing of long-shore plane slope.

The reference coordinate for Figs 7 ~ 10 is shown in Fig. 6, in which the plane slope forms a negative angle with $x$ direction and the incident water waves propagate along the opposite $y$ direction. It is obviously set to be different from the coordinate in the wave-induced long shore currents to make convenient comparison between the numerical results and the experimental data. The comparison of the pollutant concentration contours at different time between the experimental and numerical results, in which the minimum value of the pollutant concentration relative to the drainage site is set to be $10^{-3}$, is shown in Figs 7 ~ 10. It is shown in Figs 7 ~ 10 that the pollutant movement in waves and breaking-wave-induced longshore currents proceeds mainly along the shore, and the pollutant movement velocities increase with the increasing of incident wave amplitudes and the steepness of plane. Since the time for acquiring the experimental data was relatively long in one experiment case and the brightness of environmental background was frequently affected by the circumstance, it is quite difficult for quantitative analysis of pollutant concentration field in experiment. Besides, the numerical results of pollutant concentration are the averaged concentration along water column, but the experimental results of pollutant concentration are the total concentration along water column, which made the difference between the numerical and experimental results. Hence, the experimental results of pollutant concentration contours have been adopted to validate the numerical results of
Fig. 2. Comparison between the numerical and experimental results in Case 1. a. Comparison between the numerical and experimental results of wave heights, b. comparison between the numerical and experimental results of wave set-up, c. comparison between the numerical and experimental results of longshore current velocity and d. the numerical simulated longshore current velocity field. ——Numerical results, •••• experimental results.

Fig. 3. Comparison between the numerical and experimental results in Case 2. a. Comparison between the numerical and experimental results of wave heights, b. comparison between the numerical and experimental results of wave set-up, c. comparison between the numerical and experimental results of longshore current velocity and d. the numerical simulated longshore current velocity field. ——numerical results, •••• experimental results.
Fig. 4. Comparison between the numerical and experimental results in case 3. a. Comparison between the numerical and experimental results of wave heights, b. comparison between the numerical and experimental results of wave setup, c. comparison between the numerical and experimental results of longshore current velocity and d. the numerical simulated longshore current velocity field. ——Numerical results, ⋅⋅⋅ Experimental results.

Fig. 5. Comparison between the numerical and experimental results in case 4. a. Comparison between the numerical and experimental results of wave heights, b. Comparison between the numerical and experimental results of wave set-up, c. Comparison between the numerical and experimental results of long-shore current velocity, and d. the numerical simulated long-shore current velocity field. ——Numerical results, ⋅⋅⋅ experimental results.
pollutant concentration contours. It can be seen from the experimental and numerical results of the pollutant concentration contours at different time that the wave-induced currents have effectively affected the pollutant movement in coastal surf zones.

Fig. 6. The relative coordinate for pollutant movement.

Fig. 7. Comparison between the numerical and experimental results of pollutant concentration contours at different time in Case 1. a. Experimental results and b. numerical simulated results.

Fig. 8. Comparison between the numerical and experimental results of pollutant concentration contours at different time in Case 2. a. Experimental results and b. numerical simulated results.

Fig. 9. Comparison between the numerical and experimental results of pollutant concentration contours at different time in Case 3. a. Experimental results and b. numerical simulated results.
Fig. 10. Comparison between the numerical and experimental results of pollutant concentration contours at different time in Case 4. a. Experimental results and b. numerical simulated results.

4 Conclusions

Nearshore waves and wave-induced currents are mainly hydrodynamics and play important roles on pollutant movement in coastal zones. The numerical simulation of water waves, wave-induced longshore currents and movement of pollutants in waves and longshore currents have been conducted based on the hyperbolic mild slope equation, near-shore current and pollutant movement models. The numerical results have been also validated and analyzed by the experimental data. It is found that the wave-induced longshore current velocities and wave set-up increases with the increasing of incident wave amplitudes and the shore plane slope steepness; as well as the wave set-up increase with the increasing of incident wave periods; and the pollutant movement velocities also increase with the increasing of incident wave amplitudes and the shore plane slope steepness. It is concluded that the water waves and wave-induced longshore currents have effectively affected the pollutant movement in coastal surf zones.

References


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