CHGS method for numerical forecasting of typhoon waves* — I.
Spectrum of waves in growing phase

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Abstract — Owing to the fact that the wind speed and direction of typhoon vary rapidly with time and space in typhoon fetch; the nearer to the typhoon eye the greater the wind velocity, and the shorter the wind fetch the smaller the wind time, as a result, the more difficult for the wind wave to fully grow. Hence, in typhoon wave numerical calculation it is impossible to use the model for a fully grown wave spectrum. Lately, the author et al. presented a CHGS method for numerical forecasting of typhoon waves, where a model for the growing wave spectrum was set up (see Eq. (2) in the text). The model involves a parameter indicating the growing degree of wind wave, i.e., the mean wave age \( \beta \). When \( \beta \) value is small, the wave energy is chiefly concentrated near the peak frequency, so that the spectral peak gets high and steep; with the increase of \( \beta \) the spectral shape gradually gets lower and gentler; when \( \beta = 1 \), the wave fully grows, the growing spectrum becomes a fully grown P-M spectrum. The model also shows a spectral “overshooting” phenomenon within the “balance zone”.

INTRODUCTION

Since the Neumann spectrum was presented, the research over a period of more than 3 decades has clarified to a considerable extent the characteristics of the fully grown wind wave spectrum. For a fully grown wind wave, as a result of the nonlinear transporting balance of the energy among its internal component waves, the frequency distribution of the energy bears very good similarity, which greatly facilitates the mathematical description to this wave. There are a number of spectral models for the fully grown wave, among them the P-M spectrum is the most representative one. Results given by these spectral models do not differ very much from one another, and the difference is caused mainly by the representation of the data the models depended on. Owing to the complexity of real ocean wind field and wave field, the full growth could only be relative, and the regularities of full growth can only be sought through statistic average of massive observed data for the nearly fully grown ocean waves. To further study the internal structure of the ocean wave and the laws governing its generation and dying out so as to find a more concise and more theoretical spectral model of the fully grown wave, it is still one of the research subjects on ocean waves for the time being. It is well certain that a more perfect spectral model for fully grown ocean wave must be a special case of a common spectral model for

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ocean waves when both wind fetch and wind time approach maxima.

As for the spectrum of an ocean wave in growing process (shortly, "growing spectrum" hereafter), it was not possible to have a deeper study till JONSWAP's comparatively systematic and reliable observations. Up to now, the JONSWAP spectrum is still the most representative growing wave spectrum. One of the important conclusions of JONSWAP's work is; for the ocean wave within a limited wind fetch and under the action of offshore winds, its energy distribution is much more concentrated than that of the P-M spectrum, its spectral peak value is 1.5~6 times that of the P-M spectrum, with an average of 3.3 times; besides, when the wave grows with the increase of the wind fetch, the spectral maximal value obviously exceeds the energy value in the "balance zone", to which its corresponding frequency can reach in the growing process later on, i.e., the so-called energy "overshooting" phenomenon occurs. However, of the 5 parameters in the JONSWAP spectrum, only 2 vary with the wind fetch, namely, the nondimensional constant $a$ and the peak frequency $\omega_p$, while the other 3, i.e., the peak rising factor $\gamma$ and the 2 peak shape parameters $\sigma_1$ and $\sigma_2$ are with very scattered observed values and with no obvious relation to the wind fetch (Wen and Yu, 1984), so their averages are usually used. As JONSWAP carried out his observations in a range of 160 km offshore, for a bigger wind speed, the observed wave was still at a pre-growing stage, therefore, JONSWAP spectrum reflects chiefly an averaging state of the wind wave at pre-growing stage, in which the most important parameter $\gamma$ is not connected with different growing stages of the wind wave, so JONSWAP spectrum cannot continuously transit to the fully grown P-M spectrum, which brings about difficulty in application. By using the strictly selected 14 groups of data in the peripheral marine waters of Japan, Prof. Mitsuyasu gave the following expression (Mitsuyasu, 1980):

$$\gamma = 7.0F^{-1/7}.$$  

Such a work of linking $\gamma$ with the nondimensional wind fetch $F$ is quite attractive, but the data used appear too meagre.

Another experiment of Prof. Mitsuyasu's is of great significance (Mitsuyasu and Honda, 1972). In the experiment there were 3 groups A, B and C. Group A was conducted in a wind tunnel tank (20 m long, 1.8 m wide and 0.64 m from the water surface to the ceiling), Group B in an open large tank (70 m long, 8 m wide and without ceiling and side-walls), and Group C was measured at the Bodo Bay Observatory Tower with a wind fetch of 5 km or so. The nondimensional spectra of the data for these 3 groups are shown in Fig. 1. In the 2 tank subexperiments the nondimensional wind fetch was with a length $gF/U^2 = 10^2 \sim 10^3$, while that was $10^3 \sim 10^4$ in Bodo Bay. The experiment as a whole indicated that with the increase of the nondimensional wind fetch the height of the spectral peak decreased. Naturally, there was difference between the results, which was caused by different nondimensional wind fetches and their inhomogeneity.

The characteristics of a typhoon wind field are the continuous variation of wind speed and direction with space and time, in particular, in the gale fetch close to the typhoon eye, where both nondimensional wind fetch and wind time are very small, so wind waves can by no means become fully grown-up. Therefore, in the numerical calculation of typhoon wave when an ocean wave spectrum is needed, one should not use the fully grown spectrum, but the growing spectrum; otherwise, there will
be a relatively large difference between energy distributions of all frequency component waves, and the accuracy in the prediction of wave height and wave period will be affected.

![Graph showing wave spectra](image)

Fig. 1. A comparison among the nondimensional wave spectra (after Mitsuyasu, 1972).

On the basis of the knowledge of not fully grown spectra, the present study suggests a growing spectrum to treat the numerical calculation of typhoon waves. As how to use this growing spectrum in the numerical calculation of typhoon waves, another paper will be presented.

**GROWING OCEAN WAVE SPECTRUM**

Under certain conditions, regularities of the ocean wave growing with the increase of wind fetch are equivalent to those of the ocean wave growing with the increase of wind time, thus, the degree of the ocean wave growing with the wind fetch or the wind time may be described by the wave age \( \beta \). On the basis of this point and according to Sui (1984) and Voznesensky and Netsvetaev (1964), a model of growing wave spectrum is given as follows:

\[
S(\omega) = C(\beta) \frac{m_0}{\omega_p} \left( \frac{\omega}{\omega_p} \right)^{-5/\beta} \exp \left[ -1.25 \left( \frac{\omega}{\omega_p} \right)^{-4/\beta} \right],
\]

where

\[
C(\beta) = \frac{4}{\beta} (1.25)^{(\frac{\beta - 5}{4}) \Gamma^{-1}(\frac{5 - \beta}{4})},
\]

where \( \Gamma \) denotes \( \Gamma \) function. When \( \beta = 1 \),

\[
C(1) = 5,
\]

\[
S(\omega) = 5 \frac{m_0}{\omega_p} \left( \frac{\omega}{\omega_p} \right)^{-5} \exp \left[ -1.25 \left( \frac{\omega}{\omega_p} \right)^{-4} \right].
\]

Equation (4) is a P-M spectrum, namely, a fully grown wind wave spectrum. Here, it is assumed that when the ocean wave is fully grown up, \( \beta = 1 \). And \( \beta = \bar{c}/u \), is averaged wave age.

In Eq. (2) the zero order moment \( m_0 \) and the spectral peak frequency \( \omega_p \) can be substituted with the mean wave height \( H \) and the mean period \( T \) respectively.

\[
m_0 = \frac{1}{2\pi} H^2,
\]
\[
\omega_\beta = \frac{2\pi}{T}(1.25)^{\beta/4} \left[ \Gamma\left(\frac{5 - \beta}{4}\right) \right]^{\frac{1}{2}} \left[ \Gamma\left(\frac{5 - 3\beta}{4}\right) \right]^{\frac{1}{2}}.
\]

(6)

And, \( \beta \) can also be given with \( \bar{T} \) and \( u \)

\[
\beta = \frac{2\bar{T}}{u^2}.
\]

(7)

Thus, Eq. (2) can be transformed into a function of \( \bar{H} \), \( \bar{T} \) and \( u \). \( u \) is the 10 min mean wind velocity at 10 m above sea level.

In Eq. (6), when \( \beta = 1, \omega_\beta = 0.794 \frac{2\bar{T}}{\omega} \), or \( T_\omega = 1.20\bar{T} \), which is basically agreed with the statistic results of the observed data at \( T_\omega = 1.20\bar{T} \).

**CHARACTERISTICS OF THE GROWING WAVE SPECTRUM**

1. When both \( m_0 \) and \( \omega_\beta \) are constant, it can be seen from the variation of the spectral density curves of the growing spectrum with the mean wave age \( \beta \) (Fig. 2) that when the wind wave grows at the early stage the spectral energy is concentrated around the spectral peak frequency, so that the spectral peak value gets much higher than that of the P-M spectrum. With the increase of the wind fetch or wind time, the spectral peak gradually gets lower and the frequency range of energy distribution gradually gets wider; when \( \beta = 1 \), the spectrum evolves into a P-M spectrum.

2. By using an equation expressing the growth of an ocean wave (Wen and Yu, 1984) with the mean wave age \( \beta \) corresponding to wind fetch and wind time, this equation was presented by the Ocean Wave Forecasting Methods Research Group of Oceanography Panel, State Committee for Science and Technology in 1966), the growing process of a growing spectrum with wind fetch \( F \) and wind time \( t \) at wind velocity of 25 m/s is calculated and shown in Figs 3 and 4, values of the corresponding parameters are listed in Table 1, where \( H \), \( T \), and \( D \), denote the wave height, period and steepness of the significant wave, respectively. From Figs 3 and 4 the wind wave growing process described by the growing spectrum and the overshooting phenomenon occurring in the process can be seen. However, as the growing spectrum is a unipeak spectrum that does not have any nonlinear 2-order peaks, so the spectral values will not oscillate above/below the balance value. Figure 5 shows the energy overshooting occurring in the growing process of the component waves with the period of 7 s.

<table>
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<tr>
<th>( t ) (h)</th>
<th>( H_0 ) (m)</th>
<th>( T_\omega ) (s)</th>
<th>( D_\omega )</th>
<th>( \beta )</th>
<th>( F ) (km)</th>
<th>( H_0 ) (m)</th>
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Fig. 2. Nondimensional growing spectra with different \( \beta \) values.

Fig. 3. Development of a growing spectrum with wind time (Figures in the graph express wind time (h)).

Fig. 4. Development of a growing spectrum with wind fetch (Figures in the graph express wind fetch in km).
3. Figure 6 gives the nondimensional spectral shapes of a growing spectrum at different wind fetches and wind time, and its fitting with the JONSWAP spectrum. From this figure it can be seen that the earlier the growing spectrum is in the growing process, the closer to the spectral peak frequency the energy of growing spectrum is concentrated, whether it is towards high frequencies or towards low frequencies, the distribution range of energy is always narrower than that of the JONSWAP spectrum until both become fully grown up and merge into the P-M spectrum. Figure 7 shows a rough fitting of the growing spectrum and JONSWAP spectrum to the data of Groups B and C shown in Fig. 1. Viewing from the point that the energy is more concentrated around the spectral peak frequency, the growing spectrum seems to fit closer to these 2 groups of the observed data. JONSWAP spectrum was obtained from the data observed in a relatively open sea area, in spite of the fact that these data were observed under the action of the comparatively homogeneous offshore wind, yet, the wind speed and wind direction were still have varied much more than those at the Bodo Bay Observatory where the wind fetch is only 5 km long, not to say the comparison with those at the wind tunnel and the large tank. Wind waves observed at the open seas are usually mixed wind waves under the action of inhomogeneous wind field, and on the low frequency side they are unavoidably mixed with some swell component propagated from the distance, so the distributional range of the spectral energy is wider, the JONSWAP spectrum should belong to such situation.

Figure 8 gives a composite growing spectrum that is under a combined action of the main wind speed and various successively half-reduced wind speed, and its fitting to the JONSWAP spectrum. For example, the sea surface main wind speed $u_0 = 24\text{m/s}$, the successively half-reduced wind speed $u_1 = 12\text{m/s}$, $u_2 = 6\text{m/s}$, $u_3 = 3\text{m/s}$ and $u_4 = 1.5\text{m/s}$. Figure 8 is the result produced under the combined action of all these wind speeds from $u_0$ to $u_4$. Physically, it is impossible to have a combined action of various wind speeds; in assuming such a situation, the author just attempts to make the wind field in certain inhomogeneity so as to show energy distribution of the ocean wave spectrum produced by such inhomogeneous wind field. How to describe the inhomogeneity of a wind field still needs further study. Comparison between Figs 6 and 8 shows a much better fitting of the 2 spectra, which indicates that the growing spectrum can better represent the wave energy distribution in a homogeneous wind field, while the JONSWAP spectrum is better for an inhomogeneous one. Inhomogeneity of an objective wind field covers up the regularity of wave growth with wind fetch or wind time in a homogeneous
Fig. 6. A comparison between the growing spectrum and JONSWAP spectrum at different growing stages (the thick line in the graph expresses JONSWAP spectrum).
wind field, which obscures the regularity of the peak rising factor \( \gamma \) varying with wind fetch in the JONSWAP spectrum. If we can find out a parameter to describe the inhomogeneity of a wind field and its relation with parameters in JONSWAP spectrum, such as the peak rising factor \( \gamma \), prediction will be greatly improved with JONSWAP spectrum.

4. Table 2 shows the situation for parameter \( \beta \) of the growing spectrum vs. parameter \( \gamma \) of the JONSWAP spectrum at wind velocity of 10~25m/s and within the wind fetch ranging 5~300 km. The result of calculation with the data from Table 2 is similar to that obtained from Eq. (1) given in
the following:

$$\gamma = 3.44 \beta^{-4.41},$$

its correlation coefficient is \( \gamma = 0.994 \). The correlation expression of \( \gamma \) with \( \beta \) is

$$\gamma = 0.80 \beta^{-2.1},$$

its correlation coefficient is \( \gamma = 0.997 \).

<table>
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<tr>
<th>( F ) (km)</th>
<th>( u )</th>
<th>( H_c ) (m)</th>
<th>( T_c ) (s)</th>
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CONCLUDING REMARKS

Through the analysis mentioned above, it is suggested that:

1. Under the action of a certain wind velocity \( u \), in the growing process of wind wave from generation to full grown-up, the mean wave age \( \beta \) starting from certain minimum value that nearly equals to zero and gradually increasing to unity different values of \( \beta \) indicate different growing degrees of the wind wave, therefore, it should be considered reasonable to combine the wave age \( \beta \) into the growing spectral model.

2. The growing spectrum given by Eq. (2) represents the frequency distribution of the sea wave energy at different growing degree in a homogeneous wind field; the smaller the wave age \( \beta \) is, the more concentrated the energy distribution, the higher the spectral peak. Compared with the JONSWAP spectrum, the above spectrum is characterized with its energy even more concentrated around the spectral peak frequency.

3. JONSWAP spectrum fits better to the ocean wave spectrum under the action of an inhomogeneous real wind field, and its energy distribution range on both the high frequency side and on the low
frequency side of the spectrum is wider than that of the growing spectrum. This may be due to the fact that the inhomogeneity of the wind field covers up the regularity of the peak rising factor $\gamma$ varying with the wind fetch.

4. To seek a parameter that can describe the inhomogeneity of a random wind field and use it to study the relation between a random wave field and a random wind field might be an approach to improving the accuracy for calculating the sea waves.

REFERENCES


